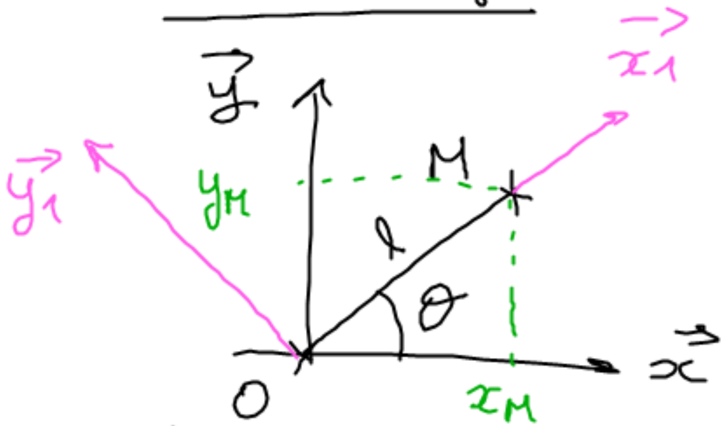
# Paramétrage




$O\vec{z}, \vec{z_1}$ $[OM] = l$

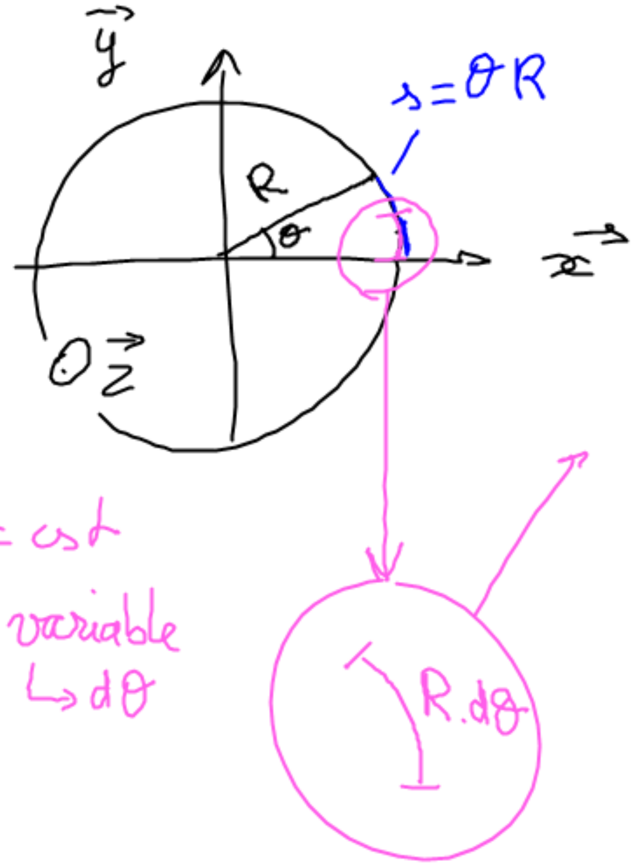
$$R = \{O, \vec{x}, \vec{y}, \vec{z}\}$$

$$R_1 = \{O, \vec{x_1}, \vec{y_1}, \vec{z_1}\}$$

$$\vec{OM} = \begin{pmatrix} x_M \\ y_M \\ 0 \end{pmatrix}_R = \begin{pmatrix} l \cdot \cos\theta \\ l \cdot \sin\theta \\ 0 \end{pmatrix}_R = \begin{pmatrix} l \\ 0 \\ 0 \end{pmatrix}_{R_1}$$

$$\vec{OM} = l\,\vec{x_1} = l\cos\theta\,\vec{x} + l\sin\theta\,\vec{y}$$

# Périmètre d'un cercle




$R = cst$

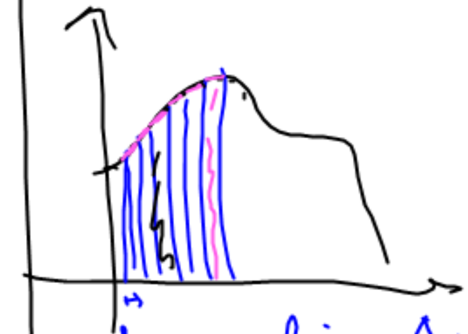
$\theta = variable$
↳ $d\theta$

$$P = \int_0^{2\pi} 1.R.d\theta = R.\int_0^{2\pi} 1.d\theta$$

$$= R.\left[\theta\right]_0^{2\pi}$$

$$= R\left(2\pi - 0\right)$$

$$\boxed{P = 2\pi.R}$$

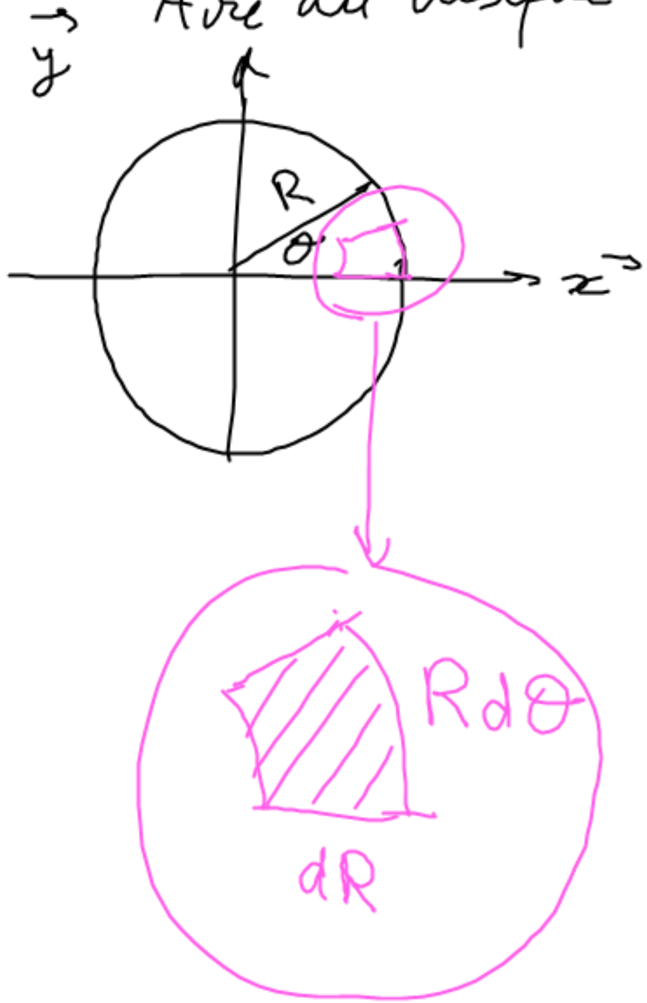



$$dx = \lim_{x \to 0^{+*}} \Delta x$$

$$\int 5x\,dx$$

$$5\int x\,dx$$

→ Aire du disque



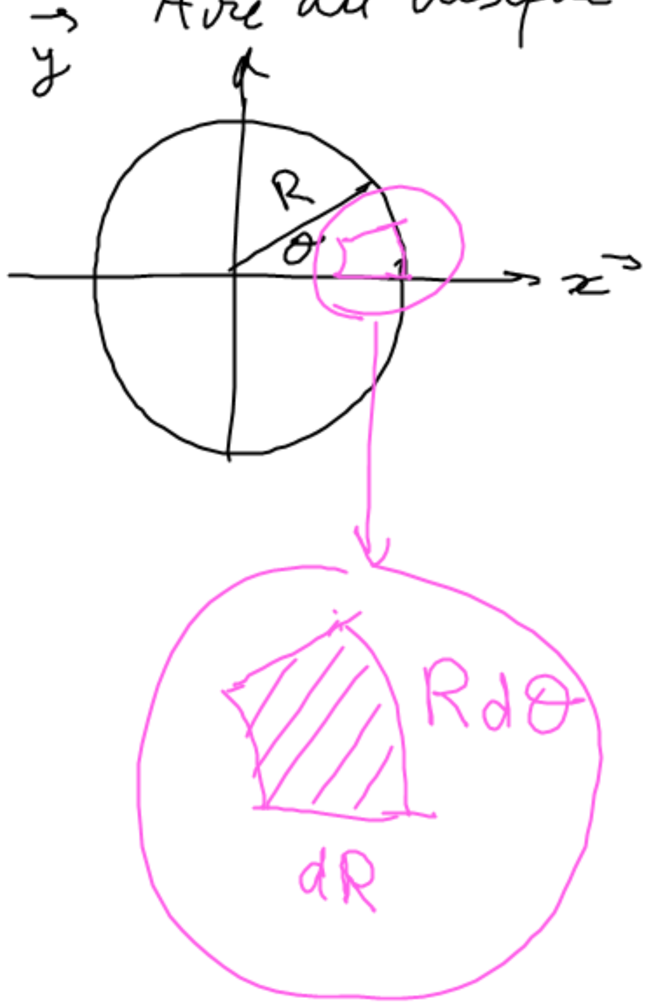

$$A = \int_0^{2\pi}\int_0^{R} R.d\theta.dR = \int_0^{2\pi} 1.d\theta . \int_0^{R} R.dR$$

$$= \left[\theta\right]_0^{2\pi} . \left[\frac{R^2}{2}\right]_0^{R}$$

$$= 2\pi . \frac{R^2}{2}$$

$$\boxed{A = \pi . R^2}$$

Aire du disque

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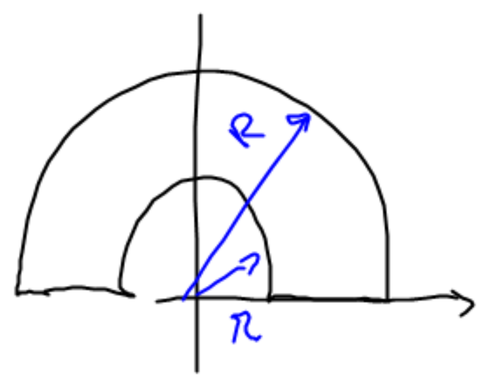
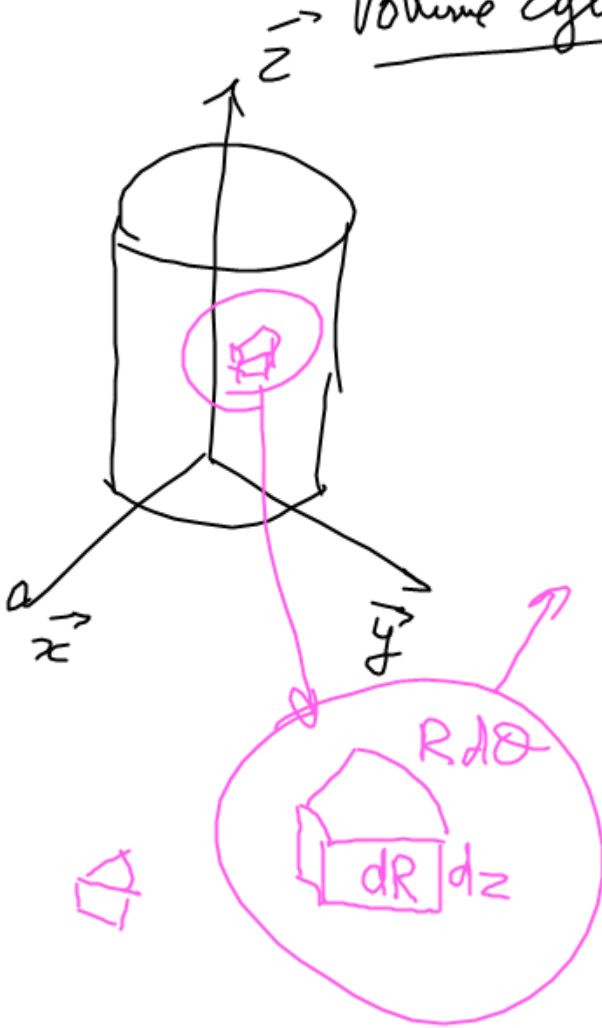



$$A = \int_0^{\pi}\int_r^{R} R d\theta dR$$

# Volume cylindre




$$V = \int_0^{2\pi}\int_0^{R}\int_0^{h} R\, d\theta\, dR\, dz$$

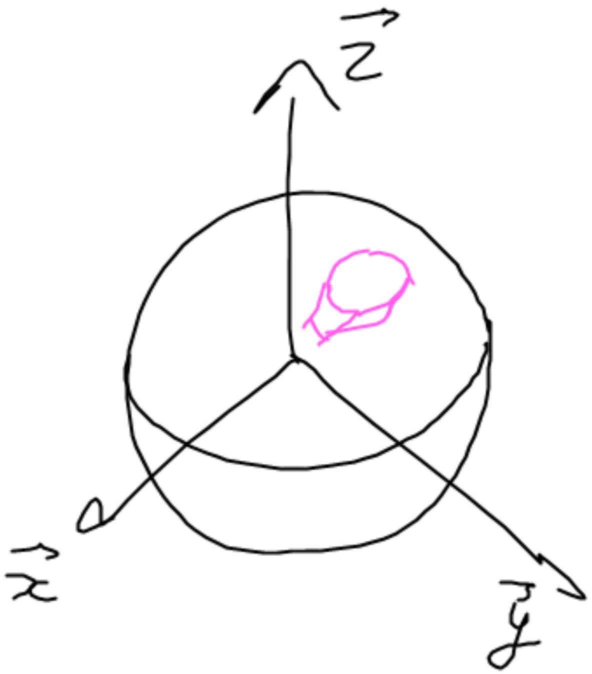
$$= \int_0^{2\pi} 1 . d\theta . \int_0^{R} R . dR . \int_0^{h} 1 . dz$$

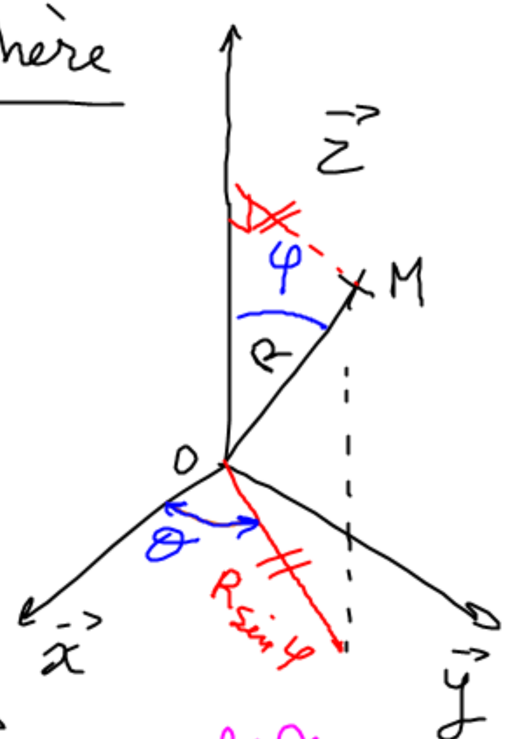
$$= \left[\theta\right]_0^{2\pi} \left[\frac{R^2}{2}\right]_0^{R} \left[z\right]_0^{h}$$

$$= 2\pi . \frac{R^2}{2} . h$$

$$\boxed{V = \pi R^2 h}$$

# Volume sphère







$$V = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} R^2 \sin\varphi \; d\theta\, d\varphi\, dR$$

$$= \int_0^{2\pi} 1 \cdot d\theta \int_0^{\pi} \sin\varphi\, d\varphi \int_0^{R} R^2\, dR$$

$$= \left[\theta\right]_0^{2\pi} \left[-\cos\varphi\right]_0^{\pi} \left[\frac{R^3}{3}\right]_0^{R}$$

$$\boxed{V = 2\pi \cdot \left(-\cos\pi - (\cos 0)\right) \cdot \frac{R^3}{3} = \frac{4\pi R^3}{3}}$$

$$(u^n)' = n \cdot u' \cdot u^{n-1}$$

$$(u \cdot v)' = uv' + u'v$$

$$(\cos u)' = -u' \sin u$$

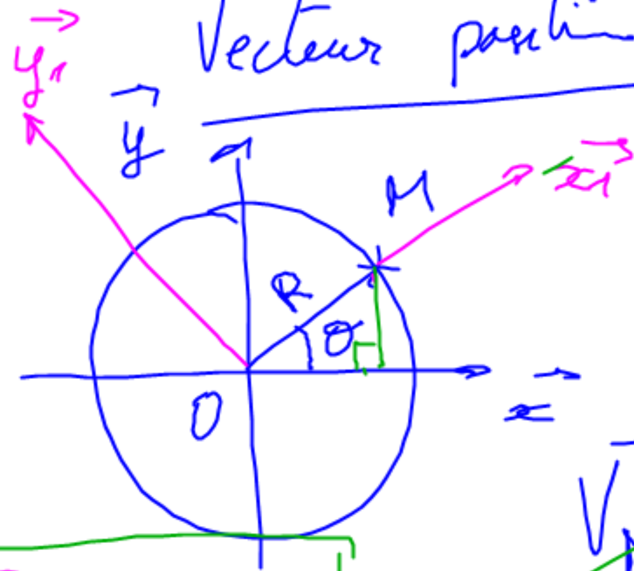
$$\vec{\Gamma} = \frac{d\vec{V}}{dt} \qquad a = \frac{dv}{dt} = \dot{v}$$

$$(\sqrt{x})' = (x^{\frac{1}{2}})'$$

$$= \frac{1}{2} \cdot 1 \cdot x^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

# Vecteur position et dérivation vectorielle




Paramétrage :

$R = cste$

$\theta = variable$

$$\vec{\Omega}_{1/0} = \begin{pmatrix} 0 \\ 0 \\ \omega_{1/0} \end{pmatrix}$$

$$\vec{OM} = \begin{pmatrix} R\cos\theta \\ R\sin\theta \\ 0 \end{pmatrix}_R = \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix}_{R_1}$$

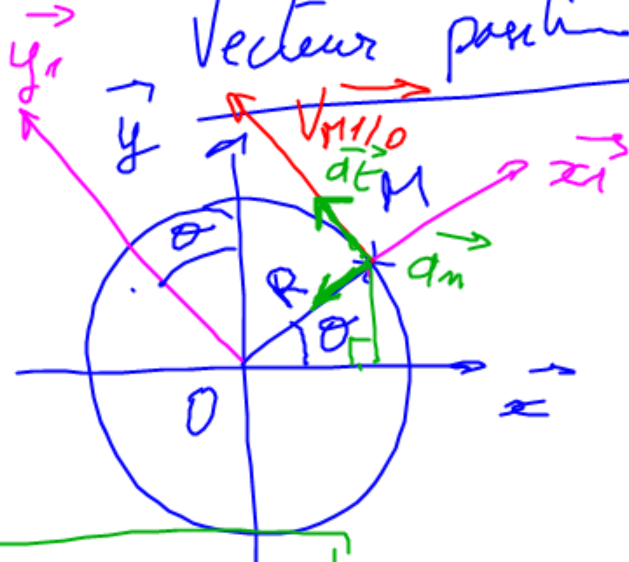
$$\vec{OM} = R\cos\theta\,\vec{x} + R\sin\theta\,\vec{y}$$

$$\vec{V}_{M1/0} = \frac{d\,\vec{OM}}{dt} = -R\dot{\theta}\sin\theta\,\vec{x} + R\dot{\theta}\cos\theta\,\vec{y}$$

$$\|\vec{V}_{M1/0}\| = \sqrt{R^2\dot{\theta}^2\underbrace{(\sin^2\theta+\cos^2\theta)}_{=1}} = \sqrt{R^2\dot{\theta}^2} = R\dot{\theta}$$

$$\boxed{V = R\omega}$$

# Vecteur position et dérivation vectorielle




**Paramétrage:**

$R = cst$

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$$\vec{\Omega}_{1/0} = \begin{pmatrix} 0 \\ 0 \\ \omega_{1/0} \end{pmatrix}$$

$$\vec{OM} = \begin{pmatrix} R\cos\theta \\ R\sin\theta \\ 0 \end{pmatrix}_R = \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix}_{R_1}$$

dérivation vectorielle

$$\boxed{\vec{V}_{M1/0} = \left[\frac{d\vec{OM}}{dt}\right]_{R_0} = \left[\frac{d\vec{OM}}{dt}\right]_{R_1} + \vec{\Omega}_{1/0} \wedge \vec{OM}}$$

$$= \left[\frac{dR\vec{x_1}}{dt}\right]_{R_1} + \omega_{1/0}\cdot\vec{z} \wedge R\cdot\vec{x_1}$$

fermeture de la main droite

$$= \vec{0} + \omega_{1/0}\cdot R\cdot(\vec{z} \wedge \vec{x_1})$$

$$\vec{V}_{M1/0} = \omega_{1/0}\cdot R\cdot\vec{y_1} = \begin{pmatrix} 0 \\ \omega_{1/0}R \\ 0 \end{pmatrix}_{R_1}$$

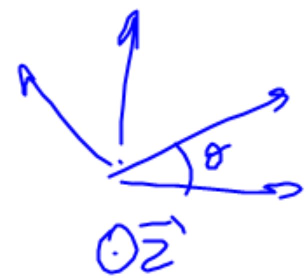
$$\|\vec{V}_{M1/0}\| = \omega_{1/0}\cdot R$$

$$\vec{z}_1 \wedge \vec{x}_1 = \vec{z}_1 \wedge \vec{x}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{R_1} \wedge \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{R_1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{R_1} = \vec{y}_1$$

$$\vec{x} \wedge \vec{x}_1 = \|\vec{x}\| . \|\vec{x}_1\| . \sin(\vec{x}, \vec{x}_1) . \vec{w}$$

vecteur unitaire "direct" normal au plan $(O, \vec{x}, \vec{x}_1)$

$$\boxed{\vec{x} \wedge \vec{x}_1 = \sin\theta \, \vec{z}}$$



$$\vec{V}_{M1/0} = \omega_{1/0} \cdot R \cdot \vec{y_1}$$

$$\vec{\Gamma}_{M1/0} = \left[\frac{d\vec{V}_{M1/0}}{dt}\right]_{R_0} = \left[\frac{d\vec{V}_{M1/0}}{dt}\right]_{R_1} + \vec{\Omega}_{1/0} \wedge \vec{V}_{M1/0}$$

$$= \left[\frac{d(\omega_{1/0} R \vec{y_1})}{dt}\right]_{R_1} + \omega_{1/0}\vec{z} \wedge \omega_{1/0} \cdot R \cdot \vec{y_1}$$

$$= \dot{\omega}_{1/0} \cdot R \vec{y_1} + \omega_{1/0}^2 \cdot R (\vec{z} \wedge \vec{y_1})$$

$$\vec{\Gamma}_{M1/0} = \underbrace{\dot{\omega}_{1/0} R}_{a_t} \vec{y_1} - \underbrace{\omega_{1/0}^2 R}_{a_n} \vec{x_1}$$